MCS PACING GUIDE & CONTENT STANDARDS FOR 7th GRADE MATHEMATICS

Fluency Expectations or Examples of Culminating Standards

- **7.EE.4**: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
    a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms ***fluently***. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
    For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards:

Significant Changes (ex: change in expectations, new Standards, or removed Standards)
7.NS.1
Slight Changes (slight change or clarification in wording)
7.SP.3
7.SP.4

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades [6-8] Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

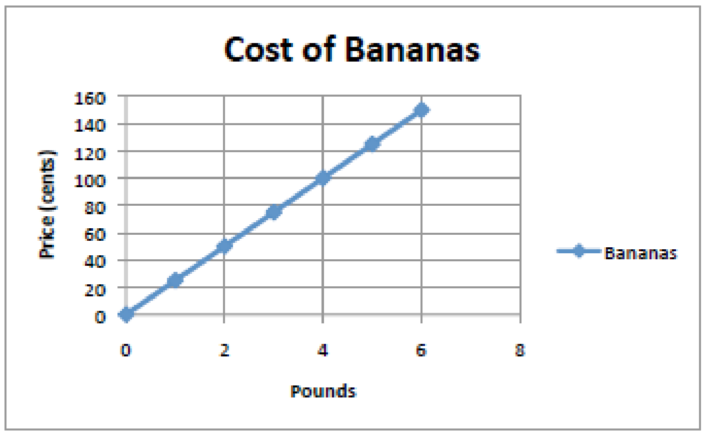
Wherever the word ***fluently*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

2016 Mississippi College- and Career-Readiness Standards for Mathematics, p. 19

Ratios and Proportional Relationships	
Cluster	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: unit rates, ratios, proportional relationships, proportions, constant of proportionality, complex fractions.	
Standard	**Clarifications**
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*	**7.RP.1** Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. The comparison can be with like or different units. Fractions may be proper or improper. Example 1: If $\frac{1}{2}$ gallon of paint covers $\frac{1}{6}$ of a wall, then how much paint is needed for the entire wall? *Solution:* $\frac{1}{2}$ gal / $\frac{1}{6}$ wall 3 gallons per 1 wall
7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	**7.RP.2** Students' understanding of the multiplicative reasoning used with proportions continues from 6th grade. Students determine if two quantities are in a proportional relationship from a table. Fractions and decimals could be used with this standard. **Note:** This standard focuses on the representations of proportions. Solving proportions is addressed in **7.SP.3**. Example 1: The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship? (see table below) *Solution:* Students can examine the numbers to determine that the price is the number of books multiplied by 3, except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the table; therefore, the table does ***not*** represent a proportional relationship.

Number of Books	Price
1	$3
3	$9
4	$12
7	$18

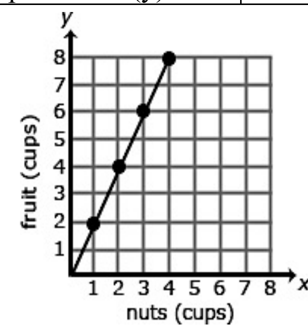
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*	Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books, 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost \$12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs. The ordered pair (1, 3) indicates that 1 book is \$3, which is the unit rate. The y-coordinate when $x = 1$ will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see previous example), graphs, equations and verbal descriptions of proportional relationships. Example 2: The graph below represents the price of the bananas at one store. What is the constant of proportionality?  *Solution:* From the graph, it can be determined that 4 pounds of bananas is \$1.00; therefore, 1 pound of bananas is \$0.25, which is the constant of proportionality for the graph. Note: Any point on the line will yield this constant of proportionality. Students write equations from context and identify the coefficient as the unit rate which is also the constant of proportionality. Example 3: The price of bananas at another store can be determined by the equation: $P = \$0.35n$, where P is the price and n is the number of pounds of bananas. What is the constant of proportionality (unit rate)? *Solution:* The constant of proportionality is the coefficient of x (or the independent variable). The constant of proportionality is 0.35.

c. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

<u>Example 4</u>: A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how the constant of proportionality was determined and how it relates to both the table and graph.

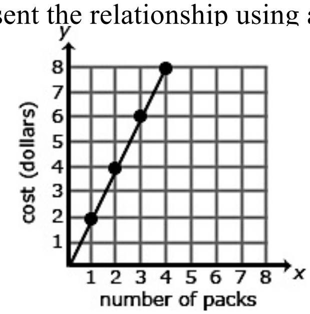
Serving Size	**1**	**2**	**3**	**4**
cups of nuts (x)	1	2	3	4
cups of fruit (y)	2	4	6	8

Solution:



The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (That is a ratio of 2:1 for cups of fruit to cups of nuts). The constant of proportionality is shown in the first column of the table and by the steepness (rate of change) of the line on the graph.

<u>Example 5</u>: The graph below represents the cost of gum packs as a unit rate of \$2 dollars for every pack of gum. The unit rate is represented as \$2/pack. Represent the relationship using a table and an equation.



Solution:

Table:

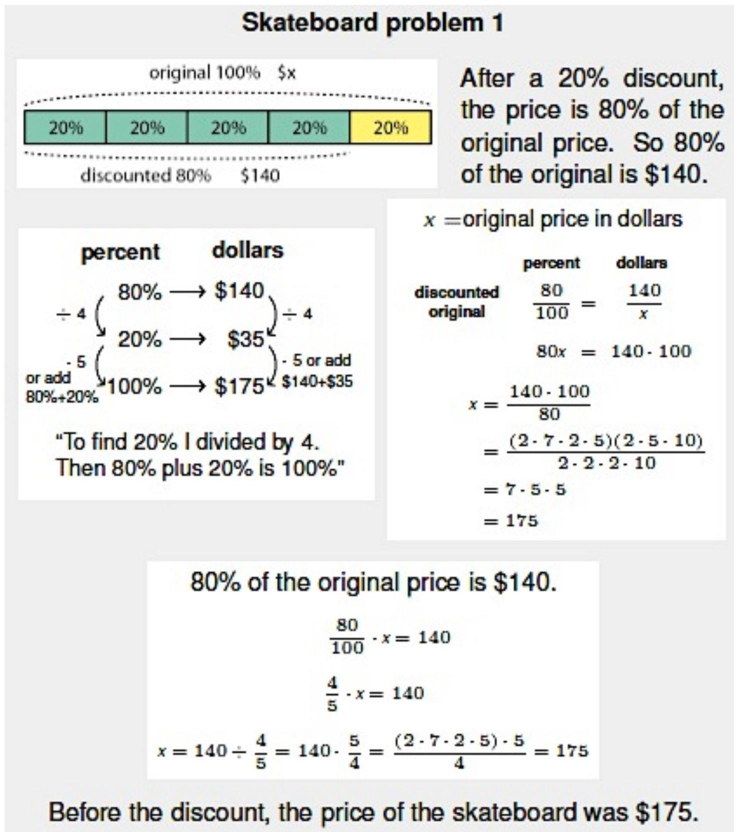
Number of Packs of Gum (g)	**Cost in Dollars (d)**
0	0
1	2
2	4
3	6
4	8

Equation: $d = 2g$, where d is the cost in dollars and g is the packs of gum

	A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as "the number of packs of gum times the cost for each pack is the total cost in dollars." They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost. (g x 2 = d) For good instructional tasks and a formative assessment lesson that support this standard, visit http://map.mathshell.org/lessons.php?unit=7215&collection=8
Cluster	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: proportion, ratio, proportional relationships, percent, simple interest, rate, principal, tax, discount, markup, markdown, gratuity, commissions, fees, percent error.	
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*	**7.RP.3** In 6th grade, students used ratio tables and unit rates to solve problems. Students expand their understanding of proportional reasoning to solve problems that are easier to solve with cross-multiplication. Students understand the mathematical foundation for cross-multiplication. Example 1: Sally has a recipe that needs $\frac{3}{4}$ teaspoon of butter for every 2 cups of milk. If Sally increases the amount of milk to 3 cups of milk, how many teaspoons of butter are needed? Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk. $\frac{3/4}{2} = \frac{x}{3}$ *Solution:* One possible solution is to recognize that $2 \bullet 1\frac{1}{2} = 3$ so $\frac{3}{4} \bullet 1\frac{1}{2} = x$. The amount of butter needed would be $1\frac{1}{8}$ teaspoons. A second way to solve this proportion is to use cross-multiplication $\frac{3}{4} \bullet 3 = 2x$. Solving for x would give $1\frac{1}{8}$ teaspoons of butter.

Students extend their work to solving multistep ratio and percent problems (**7.RP.3**). Problems involving percent increase or percent decrease require careful attention to the referent whole. For example, consider the difference in these two percent decrease and percent increase problems:

Skateboard Problem 1. After a 20% discount, the price of a SuperSick skateboard is $140. What was the price before the discount?

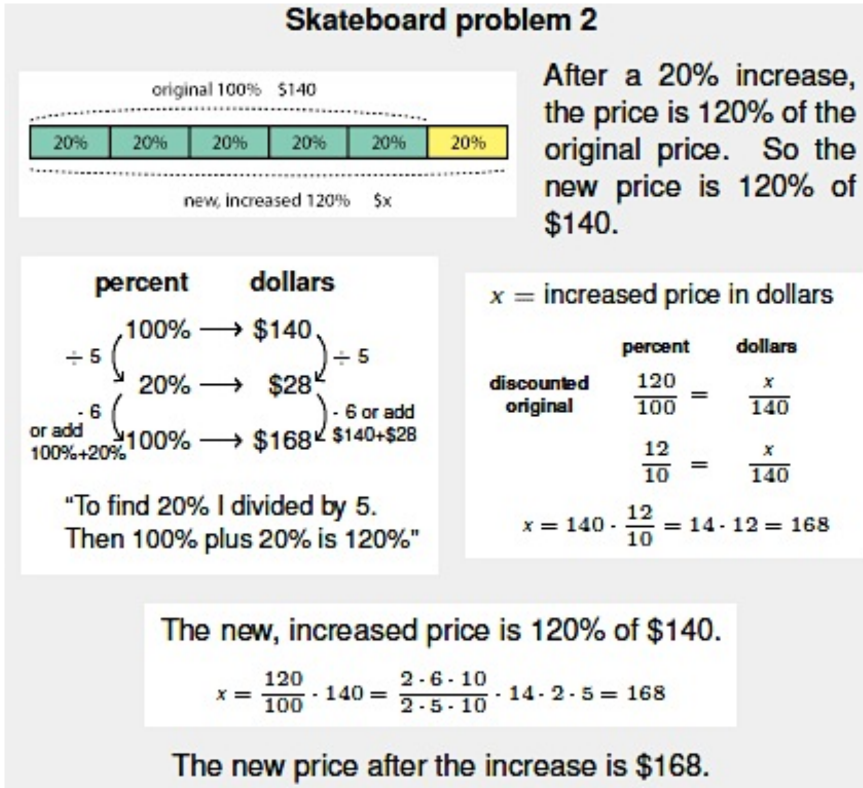


Progressions for the CCSSM (Draft): 6-7, Ratios and Proportional Relationships, December 26, 2011, p. 10

Skateboard Problem 2 on next page…

Skateboard problem 2. A SuperSick skateboard costs $140 now, but its price will go up by 20%. What will the new price be after the increase?

The solutions to these two problems are different because the 20% refers to different wholes or 100% amounts.



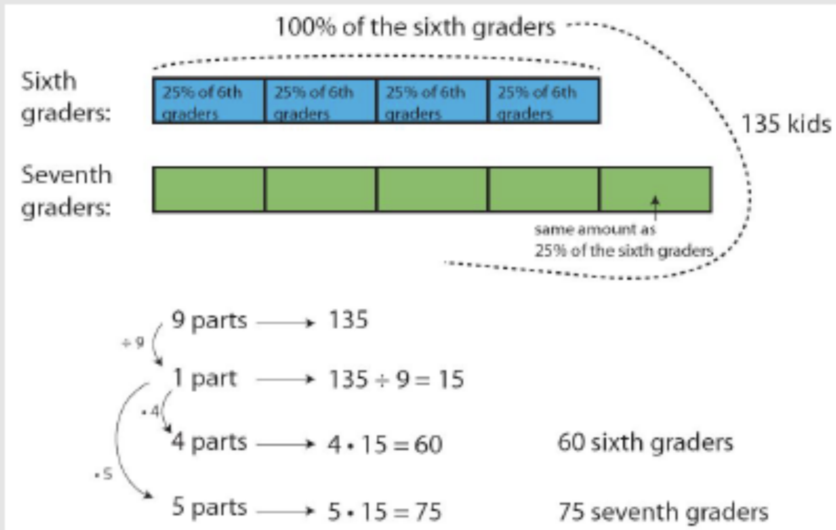
Progressions for the CCSSM (Draft): 6-7, Ratios and Proportional Relationships, December 26, 2011, p. 10

Notice that the distributive property is implicitly involved in working with percent decrease and increase. For example, in the first problem, if x s the original price of the skateboard (in dollars), then after the 20% discount, the new price is $x - 20\% \bullet x$. The distributive property shows that the new price is $80\% \bullet x$: $x - 20\% \bullet x = 100\% \bullet x - 20\% \bullet x = (100\% - 20\%)x = 80\% \bullet x$

Percentages can also be used in making comparisons between two quantities. Students must attend closely to the wording of such problems to determine what the whole or 100% amount a percentage refers to.

Using percentages in comparisons

There are 25% more seventh graders than sixth graders in the after-school club. If there are 135 sixth and seventh graders altogether in the after-school club, how many are sixth graders and how many are seventh graders?



"25% more seventh graders than sixth graders means that the number of extra seventh graders is the same as 25% of the sixth graders."

Progressions for the CCSSM (Draft): 6-7, Ratios and Proportional Relationships, December 26, 2011, p. 11

The use of proportional relationships is also extended to solve percent problems involving sales tax, markups and markdowns simple interest ($I = prt$, where I = interest, p = principal, r = rate, and t = time (in years)), gratuities and commissions, fees, percent increase and decrease, and percent error.

Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Students use models to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.

For example, Games Unlimited buys video games for \$10. The store increases their purchase price by 300%. What is the sales price of the video game?

Using proportional reasoning, if \$10 is 100% then what amount would be 300%? Since 300% is 3 times 100%, \$30 would be \$10 times 3. Thirty dollars represents the amount of increase from \$10, so the new price of the video game would be \$40.

Example 1: Gas prices are projected to increase by 124% by April 2015. A gallon of gas currently costs \$3.80. What is the projected cost of a gallon of gas for April 2015?

Solution: Possible response: "The original cost of a gallon of gas is \$3.80. An increase of 100% means that the cost will double. Another 24% will need to be added to figure out the final projected cost of a gallon of gas. Since 25% of \$3.80 is about \$0.95, the projected cost of a gallon of gas should be around \$8.55."

$$\$3.80 + 3.80 + (0.24 \text{ x } 3.80) = 2.24 \text{ x } 3.80 = \$8.512, \text{ or roughly } \$8.51$$

100%	100%	24%
\$3.80	\$3.80	?

Example 2: A sweater is marked down 33% off the original price. The original price was \$37.50. What is the sale price of the sweater before sales tax?

Solution: The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

<table>
<tr><td colspan="2">37.50
Original Price of Sweater</td></tr>
<tr><td>33% of 37.50
Discount</td><td>67% of 37.50
Sale Price of Sweater</td></tr>
</table>

Example 3: A shirt is on sale for 40% off. The sale price is \$12. What was the original price? What was the amount of the discount?

Solution:

Discount 40% of original	Sale Price → \$12 60% of original
Original Price (p)	

The sale price is 60% of the original price. This reasoning can be expressed as $12 = 0.60p$. Dividing both sides by 0.60 gives an original price of \$20.

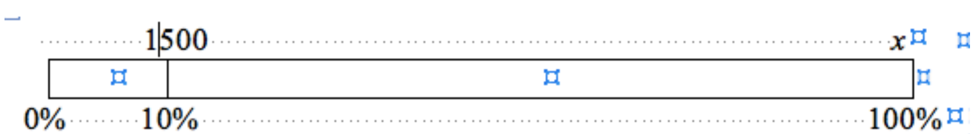
Example 4: At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs by giving all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify the solution.

Solution: The sales team members need to sell the 48 and an additional 30% of 48. 14.4 is exactly 30% so the team would need to sell 15 more TVs than in April or 63 total (48 + 15).

Example 5: A salesperson set a goal to earn \$2,000 in May. He receives a base salary of \$500 per month as well as a 10% commission for all sales in that month. How much merchandise will he have to sell to meet his goal?

Solution: \$2,000 - \$500 = \$1,500 or the amount needed to be earned as commission. 10% of what amount will equal \$1,500.

Because 100% is 10 times 10%, then the commission amount would be 10 times 1,500 or 15,000.



Example 6: After eating at a restaurant, Mr. Jackson's bill before tax is \$52.50. The sales tax rate is 8%. Mr. Jackson decides to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip Mr. Jackson leaves for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.

Solution:

The amount paid = $\underbrace{0.20 \times \$52.50}_{\text{Tip}} + \underbrace{0.08 \times \$52.50}_{\text{tax}} = 0.28 \times \$52.50$ or \$14.70

for the tip and tax. The total bill would be \$67.20,

Example 7: Stephanie paid \$9.18 for a pair of earrings. This amount includes a tax of 8%. What was the cost of the item before tax?

Solution: One possible solution path follows: \$9.18 represents 100% of the cost of the earrings + 8% of the cost of the earrings. This representation can be expressed as $1.08c = 9.18$, where c represents the cost of the earrings. Solving for c gives \$8.50 for the cost of the earrings.

Several problem situations have been represented with this standard; however, every possible situation cannot be addressed here.

The Number System

Cluster

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: rational numbers, integers, additive inverse.

7.NS.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

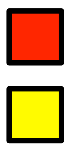
a. Describe situations in which opposite quantities combine to make 0.

For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

Teacher Note - In the 2016-17 Mississippi College- and Career-Ready Standards, the content of 7.NS.1 has been moved to 6th Grade (**6.NS.9**). However, for the 2016-17 academic year, 7th Grade teachers will still need to teach this content as current 6th Graders will not have seen it their previous year.

Students add and subtract rational numbers. Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with these operations. The expectation of the CCSS is to build on student understanding of number lines as it develops in 6th grade. Also, Algebra Tiles and Two-Colored Counters can be powerful manipulatives that can aid students in creating concrete models as they begin to discover and make sense of adding and subtracting integers as well as reinforce the concept of "opposite". This can be beneficial, as it will lay the foundation for developing concrete representations for solving equations by inverse operations.

Key:



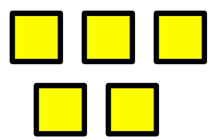
Red Tile = - 1

Yellow Tile = 1

Example 1:

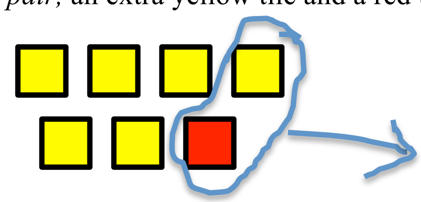
Teacher: Use your Algebra Tiles to model + 5 and using the key, draw a picture of the model you create.

Student:

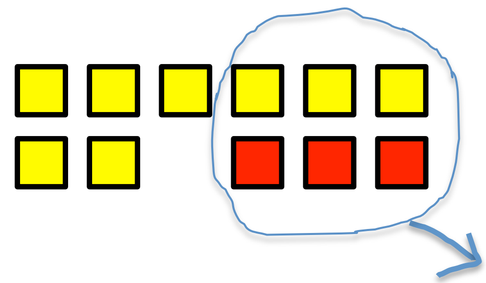


Teacher: Draw two more models that represent +5.

Student 1: My model represents + 5 because I inserted a *zero pair,* an extra yellow tile and a red tile (its opposite) to represent zero. So my model represents +5.



Student 2: My model represents + 5 because I inserted 3 *zero pairs,* 3 extra yellow tiles and 3 red tiles (an opposite for each extra yellow tile) to represent 3 sets of zero. So my model represents +5



b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

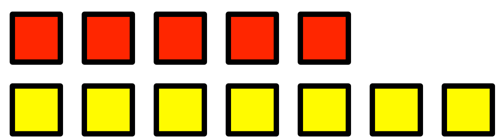
Example 2:

Teacher: Use the Algebra Tiles to solve the following problem. Draw a picture of your work.

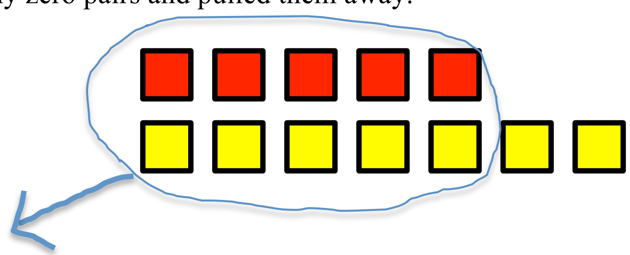
-5 + 7

Teacher: Tell me about how you solved this problem.

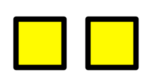
Student: First I modeled the problem by laying out 5 red tiles to represent -5 and then 7 yellow tiles to represent the action of adding +7 to my model.



Next, I grouped up my zero pairs and pulled them away.



This left me with my solution of +2.



(*Example continued on next page*.)

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

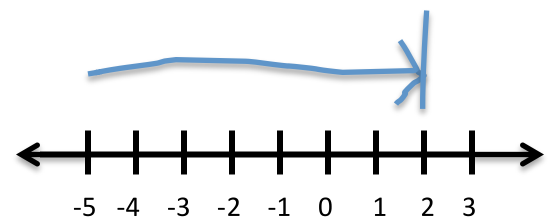
d. Apply properties of operations as strategies to add and subtract rational numbers.

Student Cont.: Even though this is an addition problem, I'm starting to notice that when I add a positive integer to a negative integer, I am "taking away" zero pairs. Before I found my solution of +2, I already knew my answer would be positive because I could clearly see that I had more yellow tiles than red tiles.

Teacher: That is a great observation! Has anyone else noticed this? Would anyone else like to share?
Students begin to discover and make sense the rules for the integer operations of addition and subtraction vs. being given meaningless rules and being asked to memorize them. For more examples of the pebble model for integer operations, see………

Example 1:

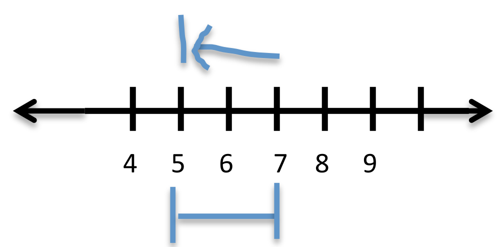
Teacher: Use a number line to add -5 + 7.



Student: I started at -5 on the number line and moved in the positive direction (to the right) 7 since I was adding 7.

The stopping point of 2 is the sum of this expression. Students also add negative fractions and decimals and interpret solutions in given contexts.

In 6th grade, students found the distance of horizontal and vertical segments on the coordinate plane. Students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line. In the example, 7 – 5, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). Therefore the answer would be 2.



Distance between 5 and 7

Example 2: Use a number line to subtract: -6 – (-4)

Solution: This problem is asking for the distance between -6 and -4. The distance between -6 and -4 is 2 and the direction from -4 to -6 is left or negative. The answer would be -2.

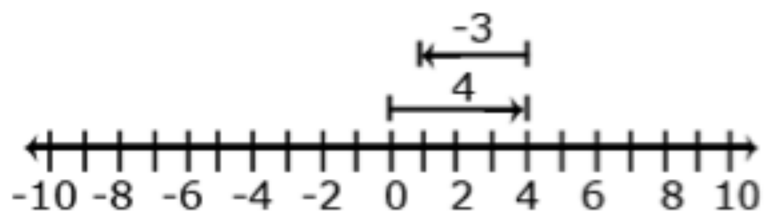
Example 3:
Use a number line to illustrate:

- *p – q ie.* 7 – 4
- *p + (-q) ie.* 7 + (– 4)
- Is this equation true *p – q = p + (-q)*?

Students explore the above relationship when *p* is negative and *q* is positive and when both *p* and *q* are negative. Is this relationship always true?

Example 4: Morgan has $4 and she needs to pay a friend $3. How much will Morgan have after paying her friend?

Solution: 4 + (-3) = 1 or (-3) + 4 = 1



7.NS.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

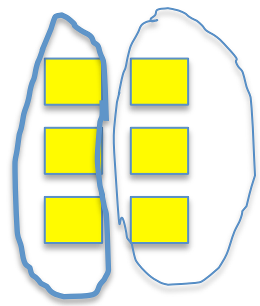
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

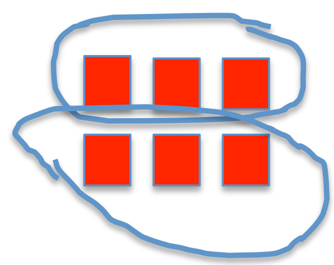
Overall, students understand that multiplication and division of integers is an extension of multiplication and division of whole numbers. Students should recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign

<u>Teacher</u>: Create a representation for the expression 2(3). Record your model and then use it to find the product.

<u>Student</u>: Okay, I think of 2(3) as "2 groups of 3." I counted out one group of three and then I counted out a second group of three. The two groups together total to 6.



<u>Teacher</u>: Create a representation for the expression 2(-3). Record your model and then use it to find the product.



<u>Student</u>: I think of 2(-3) as "2 groups of -3." I counted out one group of three negative chips and then I counted out a second group of three negative chips. The two groups together total to 6 negative chips.

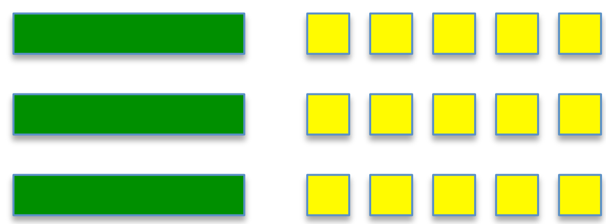
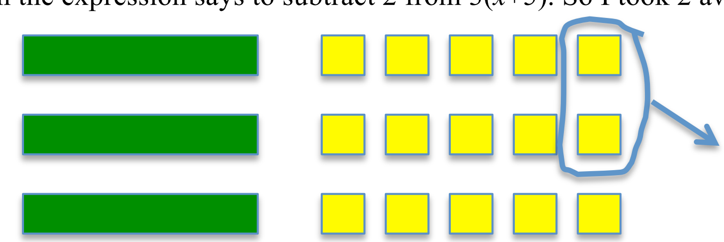
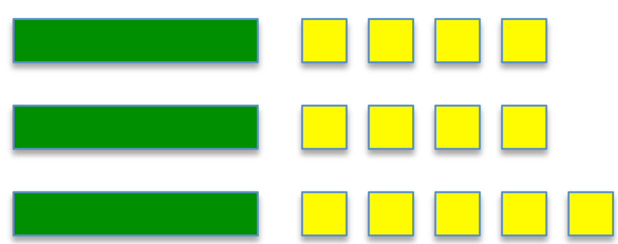
<u>Teacher</u>: Create a representation for the expression -3(2). Record your model and then use it to find the product.

<u>Student</u>: Well, I tried to represent what you asked but I don't think I can create negative groups of something. I do know that 2(-3) or two groups of negative three would give me a total of -6. The commutative property tells me the order of my factors will not change my product so, if 2(-3) = -6 then -3(2) would also have to equal -6.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for the work with rational and irrational numbers in 8th grade.
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. *Computations with rational numbers extend the rules for manipulating fractions to complex fractions.*	Students use order of operations from 6th grade to write and solve problem with all rational numbers. Example 1: Calculate: [-10(-0.9)] – [(-10) • 0.11] *Solution:* 10.1 Example 2: Jim's cell phone bill is automatically deducting \$32 from his bank account every month. How much will the deductions total for the year? *Solution:* -32 + (-32) + (-32) + (-32)+ (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) = 12 (-32) Deductions of \$384 Example 3: It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent? *Solution:* $\frac{-100 \text{ feet}}{20 \text{ seconds}} = \frac{-5 \text{ feet}}{1 \text{ second}} = -5\text{ft/sec}$ Example 4: A newspaper reports these changes in the price of a stock over four days: $\frac{-1}{8}, \frac{-5}{8}, \frac{3}{8}, \frac{-9}{8}$. What is the average daily change? *Solution:* The sum is $\frac{-12}{8}$; dividing by 4 will give a daily average of $\frac{-3}{8}$

<table>
<tr><td colspan="2">Expression and Equations</td></tr>
<tr><td colspan="2">Cluster</td></tr>
<tr><td colspan="2">Use properties of operations to generate equivalent expressions.</td></tr>
<tr><td colspan="2">Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: coefficients, like terms, distributive property, factor.</td></tr>
<tr><td>7.EE.1
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</td><td>This is a continuation of work from 6^{th} grade using properties of operations and combining like terms. Students apply properties of operations and work with rational numbers (integers and positive/negative fractions and decimals) to write equivalent expressions.

<u>Example 1</u>: Write an equivalent expression for $3(x + 5) – 2$.

<u>Student:</u> I used algebra tiles to represent the expression. First I made three groups of $(x + 5)$.

<u>Student:</u> Then the expression says to subtract 2 from $3(x+5)$. So I took 2 away from my 3 groups of $(x + 5)$.

<u>Student:</u> I am left with the solution of $3x + 13$
</td></tr>
</table>

Example 2: Suzanne says the two expressions $2(3a – 2) + 4a$ and $10a – 2$ are equivalent. Is she correct? Explain why or why not.

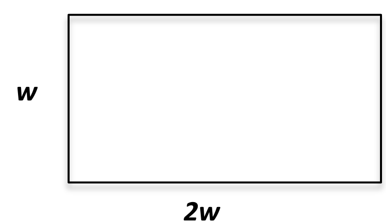
Solution:
The expressions are not equivalent. One way to prove this is to distribute and combine like terms in the first expression to get $10a – 4$, which is not equivalent to the second expression.

A second explanation is to substitute a value for the variable and perform the calculations. For example, if 2 is substituted for a then the value of the first expression is 16 while the value of the second expression is 18.

Example 3: Write equivalent expressions for: $3a + 12$.

Solution:
Possible solutions might include factoring as in $3(a + 4)$, or other expressions such as $a + 2a + 7 + 5$.

Example 4:
A rectangle is twice as long as its width. One way to write an expression to find the perimeter would be $w + w + 2w + 2w$. Write the expression in two other ways.



Solution:
$6w$ ***or*** $2(2w + w)$

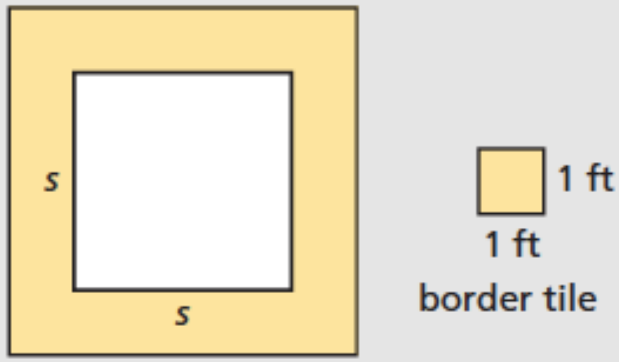
Example 5: An equilateral triangle has a perimeter of $6x + 15$. What is the length of each side of the triangle?

Solution: $3(2x + 5)$, therefore each side is $2x + 5$ units long.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*	Students understand the reason for rewriting an expression in terms of a contextual situation. For example, students understand that a 20% discount is the same as finding 80% of the cost, c ($0.80c$). Example 1: All varieties of a certain brand of cookies are \$3.50. A person buys peanut butter cookies and chocolate chip cookies. Write an expression that represents the total cost, T, of the cookies if p represents the number of peanut butter cookies and c represents the number of chocolate chip cookies *Solution:* Students could find the cost of each variety of cookies and then add to find the total. $T = 3.50p + 3.50c$ Or students could recognize that multiplying 3.50 by the total number of boxes (regardless of variety) will give the same total. $T = 3.50(p + c)$ Example 2: Jamie and Ted both get paid an equal hourly wage of \$9 per hour. This week, Ted made an additional \$27 dollars in overtime. Write an expression that represents the weekly wages of both if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week. What is another way to write the expression? Student Response 1: To find the total wage, first multiply the number of hours Jamie worked by 9. Then, multiply the number of hours Ted worked by 9. Add these two values with the \$27 overtime to find the total wages for the week. The student would write the expression $9J + 9T + 27$. Student Response 2: To find the total wages, add the number of hours that Ted and Jamie worked. Then, multiply the total number of hours worked by 9. Add the overtime to that value to get the total wages for the week. The student would write the expression $9(J + T) + 27$. Student Response 3: To find the total wages, find out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, multiply the number of hours she worked by 9. To figure out Ted's wages, multiply the number of hours he worked by 9 and then add the \$27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages. The student would write the expression $(9J) + (9T + 27)$.

Example 3:
Given a square pool as shown in the picture, write three different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression is most useful? Explain.

Writing expressions in different forms



In expressing the number of tiles needed to border a square pool with side length s feet (where s is a whole number), students might write $4(s+1)$, $s+s+s+s+4$, or $2s+2(s+2)$, each indicating a different way of breaking up the border in order to perform the calculation. They should see all these expressions as equivalent.

(Progressions for the CCSSM (Draft): 6-7, Expressions and Equations, April 22, 2011, p. 8)

<table>
<tr><td colspan="2">Cluster</td></tr>
<tr><td colspan="2">Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</td></tr>
<tr><td colspan="2">Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: numeric expressions, algebraic expressions, maximum, minimum.</td></tr>
<tr><td>7.EE.3
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Examples:

If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50.

If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 ½ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</td>
<td>Students solve contextual problems and mathematical problems using rational numbers. Students convert between fractions, decimals, and percents as needed to solve the problem. Students use estimation to justify the reasonableness of answers.

<u>Example 1</u>:
Three students conduct the same survey about the number of hours people sleep at night. The results of the number of people who sleep 8 hours a night are shown below. In which person’s survey did the most people sleep 8 hours?
• Susan reported that 18 of the 48 people she surveyed get 8 hours sleep a night
• Kenneth reported that 36% of the people he surveyed get 8 hours sleep a night
• Jamal reported that 0.365 of the people he surveyed get 8 hours sleep a night

Solution: In Susan’s survey, the number is 37.5%, which is the greatest percentage.

Estimation strategies for calculations with fractions and decimals extend from students’ work with whole number operations. Estimation strategies include, but are not limited to:
• front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts),
• clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
• rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
• using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and
• using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).</td></tr>
</table>

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms ***fluently***. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

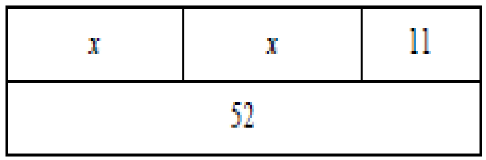
For example, the perimeter of a rectangle is 54 cm. Its length is 6cm. What is its width?

Students write an equation or inequality to model the situation. Students explain how they determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution. In contextual problems, students define the variable and use appropriate units.

Students solve multi-step equations derived from word problems. Students use the arithmetic from the problem to generalize an algebraic solution

Example 1:
The youth group is going on a trip to the state fair. The trip costs \$52. Included in that price is \$11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

Solution:
x =cost of one pass



$2x + 11 = 52$
$2x = 41$
$x = \$20.50$

Example 2: Amy had \$26 dollars to spend on school supplies. After buying 10 pens, she had \$14.30 left. How much did each pen cost including tax?

Solution:
x = number of pens
$26 = 14.30 + 10x$
Solving for x gives \$1.17 for each pen.

Example 3: The sum of three consecutive even numbers is 48. What is the smallest of these numbers?

Solution:
x = the smallest even number
$x + 2$ = the second even number
$x + 4$ = the third even number
$x + x + 2 + x + 4 = 48$
$3x + 6 = 48$
$3x = 42$
$x = 14$

<table>
<tr>
<td>b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.</td>
<td>Students solve and graph inequalities and make sense of the inequality in context. Inequalities may have negative coefficients. Problems can be used to find a maximum or minimum value when in context.

<u>Example 1:</u> Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase.

Solution:
x = cost of one t-shirt
$8x + 22 \leq 60$
$x = 4.75$, so 4 is the most t-shirts she can purchase

<u>Example 2</u>: Steven has $25 dollars to spend. He spent $10.81, including tax, to buy a new DVD. He needs to save $10.00 but he wants to buy a snack. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy?

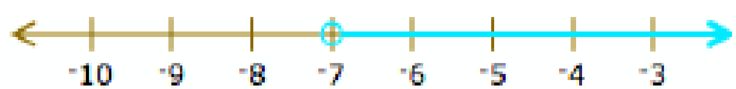
Solution:
x = number of packages of peanuts
$25 \geq 10.81 + 10.00 + 0.38x$
$x = 11.03$, so Steven can buy 11 packages of peanuts

<u>Example 3</u>: $7 - x > 5.4$

Solution: $x < 1.6$

<u>Example 4</u>: Solve $-0.5x - 5 < -1.5$ and graph the solution on a number line.

Solution: $x > -7$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Geometry

Cluster

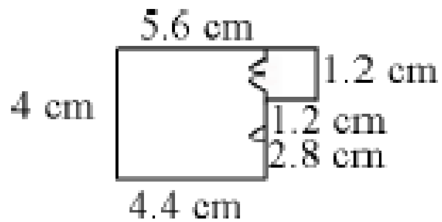
Draw, construct, and describe geometrical figures and describe the relationships between them.

Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular.

7.G.1
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

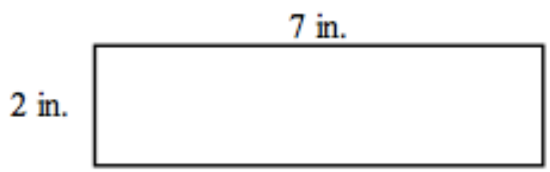
Students determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two-dimensions). Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.

Example 1: Julie shows the scale drawing of her room below. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size.



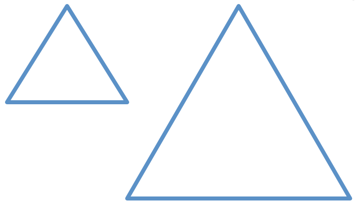
Solution:
5.6 cm → 14 ft
1.2 cm → 3 ft
2.8 cm → 7 ft
4.4 cm → 11 ft
4 cm → 10 ft

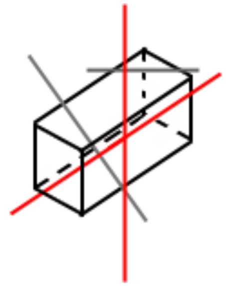
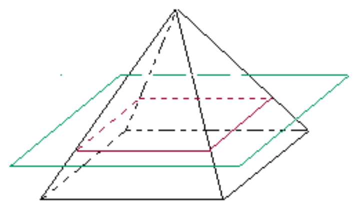
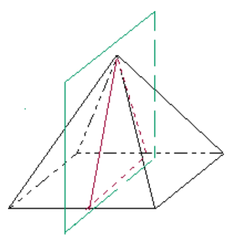
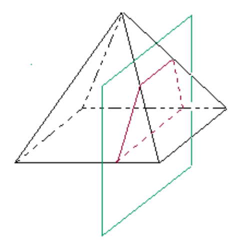
Example 2: If the rectangle below is enlarged using a scale factor of 1.5, what will be the perimeter and area of the new rectangle?



Solution:
The perimeter is linear or one-dimensional. Multiply the perimeter of the given rectangle (18 in.) by the scale factor (1.5) to give an answer of 27 in. Students could also increase the length and width by the scale factor of 1.5 to get 10.5 in. for the length and 3 in. for the width. The perimeter could be found by adding 10.5 + 10.5 + 3 + 3 to get 27 in.

The area is two-dimensional so the scale factor must be squared. The area of the new rectangle would be 14 x 1.5^2 or 31.5 in^2.

7.G.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Students draw geometric shapes with given parameters. Parameters could include parallel sides, angles, perpendicular sides, line segments, etc. Example 1: Draw a quadrilateral with one set of parallel sides and no right angles. Solution:  Students understand the characteristics of angles and side lengths that create a unique triangle, more than one triangle or no triangle. Example 2: Can a triangle have more than one obtuse angle? Explain your reasoning. Student: The sum of the interior angles of a triangle is 180 degrees. Obtuse means greater than 90 degrees. If you tried to have more than one obtuse angle, you'd have more than two angles worth 90 degrees – which would be more than 180 degrees. You'd run out of room. So, no you can't have more than one obtuse angle. Example 3: Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not? Student: You can draw more than one triangle that has all 60 degree angles. You could have all kinds of sizes, as long as all the angles measure the same. I drew one big one and one little one, just to show.  Example 4: Will three sides of any length create a triangle? Explain how you know which will work. Possibilities to examine are: a. 13 cm, 5 cm, and 6 cm b. 3 cm, 3cm, and 3 cm c. 2 cm, 7 cm, 6 cm *Solution:* "A" above will not work; "B" and "C" will work. Students recognize that the sum of the two smaller sides must be larger than the third side.

<table>
<tr>
<td>7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</td>
<td>7.G.3 Students need to describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids. Cuts made parallel will take the shape of the base; cuts made perpendicular will take the shape of the lateral (side) face. Cuts made at an angle through the right rectangular prism will produce a parallelogram.

If the pyramid is cut with a plane (green) parallel to the base, the intersection of the pyramid and the plane is a square cross section (red).

If the pyramid is cut with a plane (green) passing through the top vertex and perpendicular to the base, the intersection of the pyramid and the plane is a triangular cross section (red).


If the pyramid is cut with a plane (green) perpendicular to the base, but not through the top vertex, the intersection of the pyramid and the plane is a trapezoidal cross section (red).
http://intermath.coe.uga.edu/dictnary/descript.asp?termID=95</td>
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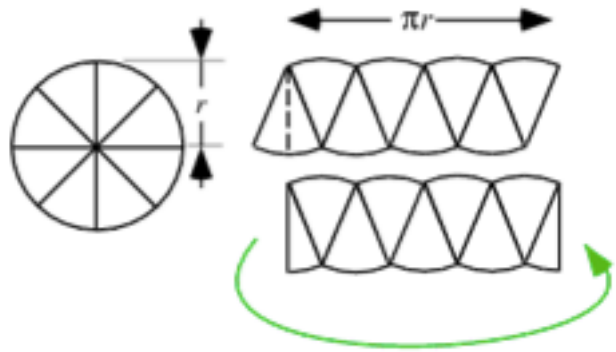
<table>
<tr><th colspan="2">Cluster</th></tr>
<tr><td colspan="2">Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</td></tr>
<tr><td colspan="2">Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: inscribed, circumference, radius, diameter, pi, supplementary, vertical, adjacent, complementary, pyramids, face, base</td></tr>
<tr><td>7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</td><td>7.G.4 Students understand the relationship between radius and diameter. Students also understand the ratio of circumference to diameter can be expressed as π. Building on these understandings, students generate the formulas for circumference and area.

The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown, a parallelogram results. Half of an end wedge can be moved to the other end and a rectangle results. The height of the rectangle is the same as the radius of the circle. The base length is $\frac{1}{2}$ the circumference ($2\pi r$). The area of the rectangle (and therefore the circle) is found by the following calculations:



http://mathworld.wolfram.com/Circle.html

A rect = Base x Height
A r e a = . $(2\pi r)$ x r
A r e a = πr x r
Area = πr^2

Students solve problems (mathematical and real-world) involving circles or semi-circles.

Note: Because π is an irrational number that neither repeats nor terminates, the measurements are approximate when 3.14 is used in place of Π.</td></tr>
</table>

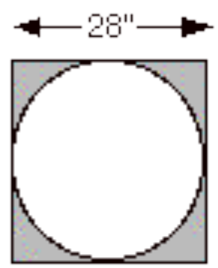
Example 1:
The seventh grade class is building a mini-golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might someone communicate this information to the salesperson to make sure he receives a piece of carpet that is the correct size? Use 3.14 for π.

Solution:
Area = πr^2
Area = 3.14 $(5)^2$
Area = 78.5 ft^2

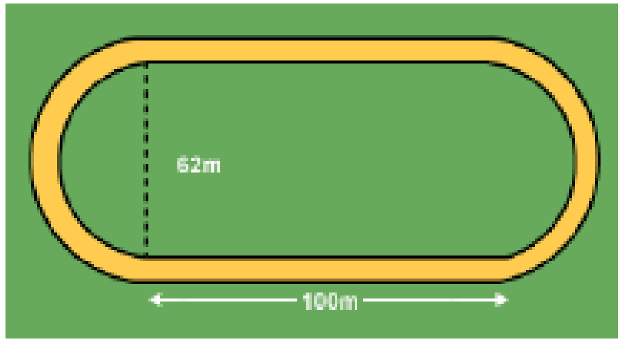
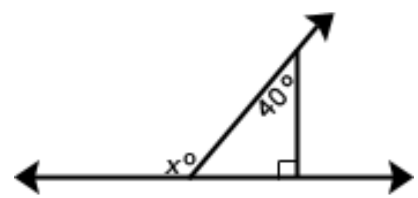
To communicate this information, ask for a 9 ft by 9 ft square of carpet. A circle of that size could be cut out of a square measuring 81 square feet in area.

Students build on their understanding of area from 6^{th} grade to find the area of leftover materials when circles are cut from squares and triangles or when squares and triangles are cut from circles.

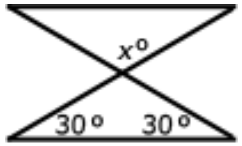
Example 2:
If a circle is cut from a square piece of plywood, how much plywood would be left over?



Solution:
The area of the square is 28 x 28 or 784 in^2. The diameter of the circle is equal to the length of the side of the square, or 28", so the radius would be 14". The area of the circle would be approximately 615.44 in^2. The difference in the amounts (plywood left over) would be 168.56 in^2 (784 – 615.44).

	 Example 3: What is the perimeter of the inside of the track above? *Solution:* The ends of the track are two semicircles, which would form one circle with a diameter of 62m. The circumference of this part would be 194.68 m. Add this to the two lengths of the rectangle and the perimeter is 394.68 m. ***<u>TEACHER NOTE</u>***: “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area and circumference) and the figure. This understanding should be for *all* students through multiple experiences with different types of problems.
7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	Students use understandings of angles and deductive reasoning to write and solve equations Example1: Write and solve an equation to find the measure of angle *x*.  *Solution:* Find the measure of the missing angle inside the triangle (180 – 90 – 40), or 50°. The measure of angle *x* is supplementary to 50°, so subtract 50 from 180 to get a measure of 130° for *x*.

Example 2:
Find the measure of angle *x*.



Solution:
First, find the missing angle measure of the bottom triangle (180 – 30 – 30 = 120). Since the 120 is a vertical angle to *x*, the measure of *x* is also 120°.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

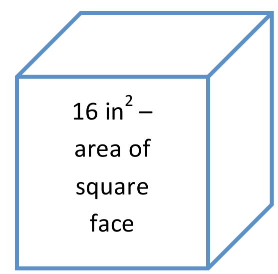
7.G.6 Students continue work from 5th and 6th grade to work with area, volume, and surface area of two-dimensional and three-dimensional objects (composite shapes). Students will not work with cylinders, as circles are not polygons. At this level, students determine the dimensions of the figures given the area or volume.

TEACHER NOTE: “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for *all* students.

Surface area formulas are not the expectation in 7th Grade. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.

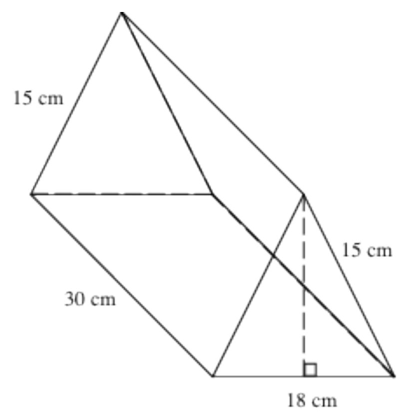
Students’ understanding of volume can be supported by focusing on the area of base times the height to calculate volume. Students solve for missing dimensions, given the area or volume.

Example: The surface area of a cube is 96 in^2. What is the volume of the cube?



Student: A cube has 6 faces that are all squares. All the squares are the same size. Dividing 96 by 6 gives me 16. So now I know each square has an area of 16 in^2. To find the volume of the cube, I need to know the length of each edge of the cube. Each edge is the same as the side length of one of the squares. 4×4 is 16, so, the side of the square is 4 inches. Because all of the edge lengths are the same in a cube, I can do $4 \times 4 \times 4 = 64$ in^3.

Example 5:
Jennie purchased a box of crackers from the deli. The box is in the shape of a triangular prism (see diagram below). If the volume of the box is 3,240 cubic centimeters, what is the height of the triangular face of the box? How much packaging material was used to construct the cracker box? Explain how you got your answer.



Solution:
Volume can be calculated by multiplying the area of the base (triangle) by the height of the prism. Substitute given values and solve for the area of the triangle.

$V = Bh$
$3{,}240\ \text{cm}^3 = B\ (30\text{cm})$
$\dfrac{3{,}240\ \text{cm}^3}{30\ \text{cm}} = \dfrac{B(30\text{cm})}{30\ \text{cm}}$

$108\ \text{cm}^2 = B$ (area of the triangle)

To find the height of the triangle, use the area formula for the triangle, substituting the known values in the formula and solving for height. The height of the triangle is 12 cm.

The problem also asks for the surface area of the package. Find the area of each face and add:
2 triangular bases: ½ (18 cm)(12 cm) = 108 cm^2 x 2 = 216 cm^2
2 rectangular faces: 15 cm x 30 cm = 450 cm^2 x 2 = 900 cm^2
1 rectangular face: 18 cm x 30 cm = 540 cm^2

Adding 216 cm^2 + 900 cm^2 + 540 cm^2 gives a total surface area of 1656 cm^2.

Statistics and Probability

Cluster

Use random sampling to draw inferences about a population

Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: random sampling, population, representative sample, inferences.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences

7.SP.1 Students recognize that it is difficult to gather statistics on an entire population. Instead a random sample can be representative of the total population and will generate valid predictions. Students use this information to draw inferences from data. A random sample must be used in conjunction with the population to get accuracy. For example, a random sample of elementary students cannot be used to give a survey about the prom.

Example 1:
The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students' preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Determine if each survey option would produce a random sample. Which survey option should the student council use and why?

1. Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey.
2. Survey the first 20 students that enter the lunchroom.
3. Survey every 3rd student who gets off a bus.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

7.SP.2 Students collect and use multiple samples of data to make generalizations about a population. Issues of variation in the samples should be addressed.

Example 1:
Below is the data collected from two random samples of 100 students regarding students' school lunch preference. Make at least two inferences based on the results.

Student Sample	Hamburgers	Tacos	Pizza	Total
#1	12	14	74	100
#2	12	11	77	100

Solution:

- Most students prefer pizza.
- More people prefer pizza than hamburgers and tacos combined.

<table>
<tr><td colspan="2">Cluster</td></tr>
<tr><td colspan="2">Draw informal comparative inferences about two populations.</td></tr>
<tr><td colspan="2">Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: variation/variability, distribution, measures of center, measures of variability.</td></tr>
<tr><td>7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability <s>(mean absolute deviation)</s> on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</td><td>For the 2016-17 Mississippi College- and Career-Readiness Standards, a review committee removed “mean absolute deviation” as an expectation within this standard.

This is the students’ first experience with comparing two data sets. Students build on their understanding of graphs, mean, median, and interquartile range from 6th grade. Students understand that…
1. A full understanding of the data requires consideration of the measures of variability as well as mean or median.
2. Variability is responsible for the overlap of two data sets and that an increase in variability can increase the overlap.
3. Median is paired with the interquartile range and mean is paired with the mean absolute deviation.</td></tr>
<tr><td>7.SP.4 Use measures of center and measures of variability (i.e., interquartile range) for numerical data from random samples to draw informal comparative inferences about two populations.

For example, decide whether</td><td>Students compare two sets of data using measures of center (mean and median) and variability MAD and IQR).

Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.

(Example continued on next page.)</td></tr>
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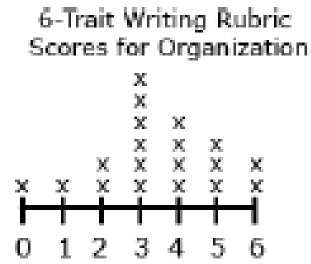
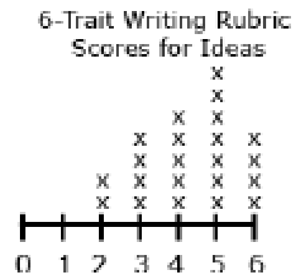
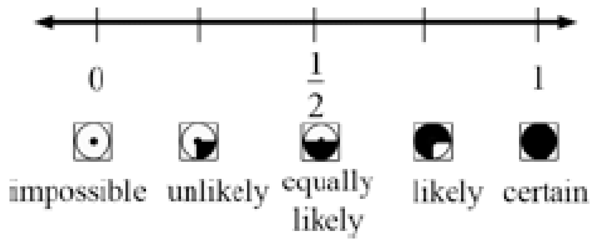
the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	  Example 1: The two data sets below depict random samples of the management salaries in two companies. Based on the salaries below which measure of center will provide the most accurate estimation of the salaries for each company? • Company A: 1.2 million, 242,000, 265,500, 140,000, 281,000, 265,000, 211,000 • Company B: 5 million, 154,000, 250,000, 250,000, 200,000, 160,000, 190,000 *Solution:* The median would be the most accurate measure since both companies have one value in the million that is far from the other values and would affect the mean.

Cluster
Investigate chance processes and develop, use, and evaluate probability models.
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: sample spaces.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

TEACHER NOTE: This is the students' first formal introduction to probability in the standards..

Students recognize that the probability of any single event can be can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1, inclusive, as illustrated on the number line below.



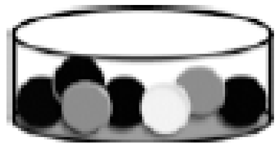
The closer the fraction is to 1, the greater the probability the event will occur. Larger numbers indicate greater likelihood. For example, if someone has 10 oranges and 3 apples, you have a greater likelihood of selecting an orange at random.

Students also recognize that the sum of all possible outcomes is 1.

Example 1:
There are three choices of jellybeans – grape, cherry, and orange. If the probability of getting a grape is $\frac{3}{10}$ and the probability of getting cherry is $\frac{1}{5}$, what is the probability of getting orange?

Solution:
The combined probabilities must equal 1. The combined probability of grape and cherry is $\frac{5}{10}$. The probability of orange must equal $\frac{5}{10}$ to get a total of 1.

Example 2:
The container below contains 2 gray, 1 white, and 4 black marbles. Without looking, if Eric chooses a marble from the container, will the probability be closer to 0 or to 1 that Eric will select a white marble? A gray marble? A black marble? Justify each of your predictions.



Solution:
White marble: Closer to 0
Gray marble: Closer to 0
Black marble: Closer to 1

Students can use simulations such as Marble Mania on AAAS or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns.

Marble Mania http://www.sciencenetlinks.com/interactives/marble/marblemania.html
Random Drawing Tool **-** http://illuminations.nctm.org/activitydetail.aspx?id=67

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

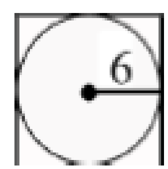
7.SP.6 Students collect data from a probability experiment, recognizing that as the number of trials increase, the experimental probability approaches the theoretical probability.

The focus of this standard is relative frequency --
The relative frequency is the observed number of successful events for a finite sample of trials. Relative frequency is the observed proportion of successful event, expressed as the value calculated by dividing the number of times an event occurs by the total number of times an experiment is carried out.

Example 1:
Suppose we toss a coin 50 times and have 27 heads and 23 tails. We define a head as a success. The relative frequency of heads is:

$\frac{27}{50}$= 54%

The probability of a head is 50%. The difference between the relative frequency of 54% and the probability of 50% is due to small sample size.

	The probability of an event can be thought of as its long-run relative frequency when the experiment is carried out many times. Students can collect data using physical objects or graphing calculator or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies. Example 2: Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. They make conjectures about theoretical probabilities (How many green draws would be expected if 100 pulls are conducted?) Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities as groups and then as a whole class to compare how the greater number of samples (i.e., "whole class") affects how closely the experimental probability reflects the theoretical probability. Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. (An example would be 3 green marbles, 6 blue marbles, 3 blue marbles.) Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability.
7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a*	**7.SP.7** Probabilities are useful for predicting what will happen over the long run. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size. Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data. Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects or graphing calculator or web-based simulations. Students can also develop models for geometric probability (i.e. a target). Example 1: If Mary chooses a point in the square, what is the probability that it is not in the circle?  *Solution:* The area of the square would be 12 x 12 or 144 units squared. The area of the circle would be 113.04 units squared. The probability that a point is not in the circle would be $\frac{30.96}{144}$ or 21.5%

class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	Example 2: Jason is tossing a fair coin. He tosses the coin ten times and it lands on heads eight times. If Jason tosses the coin an eleventh time, what is the probability that it will land on heads? *Solution:* The probability would be $\frac{1}{2}$. The result of the eleventh toss does not depend on the previous results. Example 3: Devise an experiment using a coin to determine whether a baby is a boy or a girl. Conduct the experiment ten times to determine the gender of ten births. How could a number cube be used to simulate whether a baby is a girl or a boy or girl? Example 4: Conduct an experiment using a Styrofoam cup by tossing the cup and recording how it lands. • How many trials were conducted? • How many times did it land right side up? • How many times did it land upside down/ • How many times did it land on its side? • Determine the probability for each of the above results
7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	Students use tree diagrams, frequency tables, and organized lists, and simulations to determine the probability of compound events. Example 1: How many ways could the 3 students, Amy, Brenda, and Carla, come in 1^{st}, 2^{nd} and 3^{rd} place? *Solution:* Making an organized list will identify that there are 6 ways for the students to win a race A, B, C A, C, B B, C, A B, A, C C, A, B C, B, A

b. Represent for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	Example 2: A fair coin will be tossed three times. What is the probability that two heads and one tail in any order will result? (Adapted from SREB publication *Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do*) *Solution:* First, let's find all of the possible outcomes of tossing a coin three times. H will represent heads; T will represent tails. All possible outcomes: HHH HHT HTH THH HTT TTH HTH TTT How many of those possible outcomes represent 2 Heads and 1 Tail? HHT, HTH and THH So the probability would be $\frac{3}{8}$.

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